A STUDY OF SAVINGS FROM X^2-DISTRIBUTED INCOME

By

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Introduction :

Methodological studies on savings are entirely lacking. An attempt is made in this paper to study stable behaviour of savings pattern for some types of income distribution. The motivation for the construction of savings model is based on its similarity with the contents of a dam. In this paper stable distribution of savings is studied when incomes follow a x^2-distribution with 'n' degrees of freedom. As the degrees of freedom are changing the change in the stable probability distribution of savings is studied.

Dam Model :

Stochastic model for dam contents was first proposed by P.A.P. Moran (1954), his model may be stated as : 'For a finite dam of capacity 'K' units, let x_{t-1} be the random inflow (or input of water at the beginning of the period $(t-1, t)$, and let z_{t-1} be the dam contents just before the inflow of x_{t-1}. Let p_i be the probability for the input of water to be 'i' units (i=0, 1, 2,......). Let 'M' be a suitable fixed positive integer, and let the release of water after inflow of x_{t-1} but before the end of the period $(t-1, t)$ be the minimum of the two numbers $Z_{t-1}+X_{t-1}$ and M. Let it be denoted by Y_{t-1}. Then the residual dam contents at the beginning of the period $(t, t+1)$ are

$$Z_t=Z_{t-1}+X_{t-1}-Y_{t-1}$$

If P_g is the probability for Z_t to be 'g' units, then the equations for the determination P's in terms of P's are

$$P_0=P_0\,(p_0+p_1+\ldots\ldots p_M)+P_1(p_0+\ldots p_{M-}\ )+\ldots P_M p_0$$

$$P_1 = P_0\,(p_{M+1}) + P_1 p_M + \ldots\ldots P_{M+1}\, p_0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$P_{K-M} = P_0(p_k + \ldots\ldots) + P_1(p_{K-1} + \ldots\ldots) + \ldots + P_{K-M}(p \;\; + \ldots)$$

It is clear that

$$P_{K-M+1} = P_{K-M+2} = \ldots\ldots = \ldots\ldots = 0.$$

Taking Moran's work as a basis, some futher work is done by D.G. Kendall (1957), H.E. Daniels, J. Gani (1955), N.V. Prabhu (1959), K. Naghabhushanam and M. Perayya Sastry (1962) and others.

SAVING MODEL :

Considering the similarities between cumulative savings and Dam contents a Stochastic model for savings was proposed by K. Naghabhushanam and M. Perayya Sastry (5) as given below :

Let R_0, $R_1 \ldots\ldots R_k$ be an increasing sequence of $k+1$ positive integers with $R_0 = 0$ and let M be a positive integer.

Then

$$Y_t = X_t + Z_t \text{ if } X_{t-1} + Z_{t-1} < M$$

$$= X_{t-1} + Z_{t-1} - (j+1) \text{ if } M + R_j \leqslant X_{t-1} + Z_{t-1} < M + R_{j+1}$$

$$j = 0, 1, \ldots\ldots (k-1)$$

Let us suppose that persons with income below M units cannot save any amount. We also suppose that persons in income group $(M, M+R_1)$ save one unit per year, those in income range $(M+R_1, M+R_2)$ save two units per year and so on. Let us suppose that incomes can go upto $M+R_k$, so that maximum savings is k units.

Using the above mentioned release rule (Savings release rule) Savings equations can be given as

$$P_o = P_o(p_o + \ldots\ldots p_M) + P_1(p_o + \ldots\ldots p_{M-1}) + \ldots\ldots P_k(p_o + \ldots\ldots p_{M-k})$$

$$P_1 = P_e(p_{M+1} + \cdots p_{M+R_1}) + P_1(p_M + \cdots p_{M+R_1-1}) \ldots\ldots$$

$$\ldots\ldots + P_k(p_{M-k+1} + \ldots\ldots p_{M+R_1 k-})$$

... ...

... ...

... ...

$$P_k = P_o(p_{M+R_{k-1}+1} + P_{M+R_k}) + P_k(p_{M+R_{k-1}-k} + \cdots p_{M+R_1-k})$$

It is clear that

$$P_{k+1} = P_{k+2} = \dots\dots\dots = 0.$$

In the savings equations discrete distribution of incomes is considered. If a continous income distribution is taken, then we get savings equation in the form of integral equations.

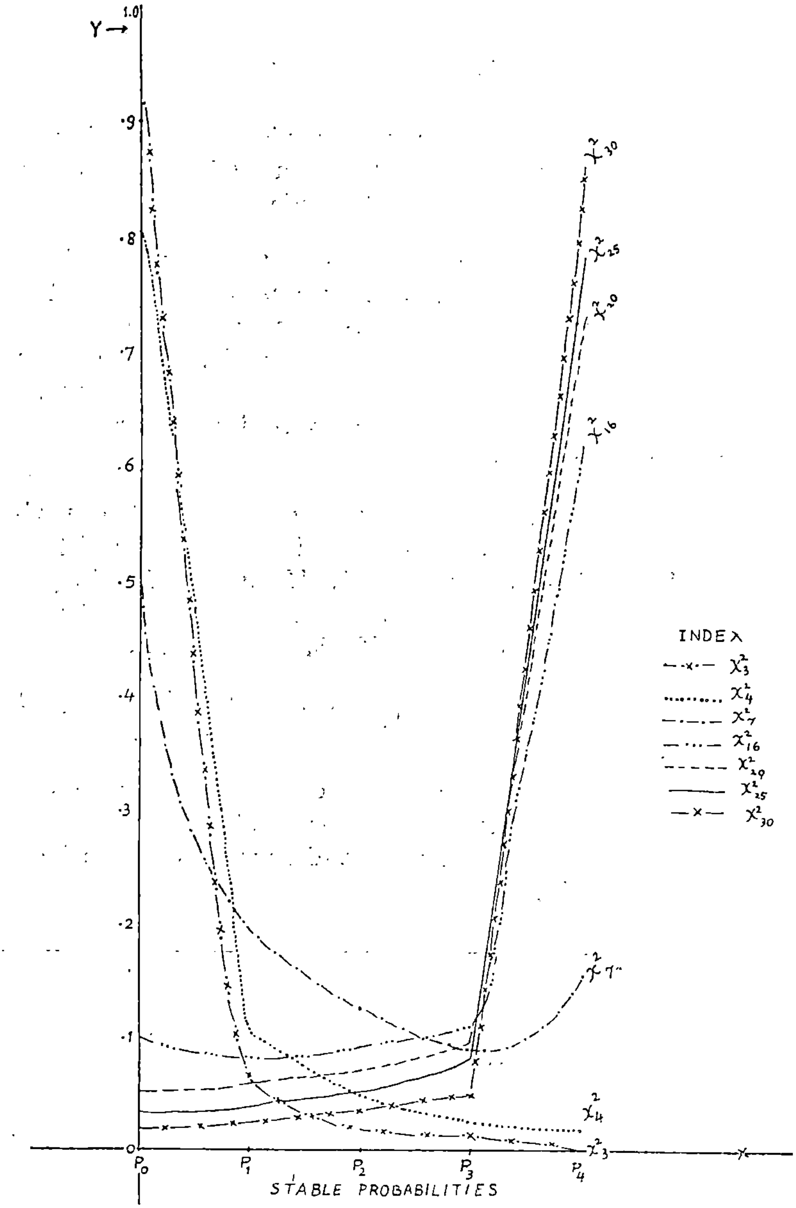
So when a continuous uni-modal distribution is proposed for incomes, and tables are not available, a discrete approximation has to be worked out for adopting the method given in this paper. In such a case, a two-parameter approximation given by one of the authors, M.P. Sastry (1960) can be used with advantage. To illustrate the method, such an approximation is worked out inspite of the availability of tables for chi-square.

Savings equations are derived assuming incomes follow a x^2-distribtion with $n=2, 3, 4, \dots\dots 30$, when the income groups are defined as (O, M), $(M+1,\ M+R_1)$, $(M+R_1+1,\ M+R_2)\dots\dots(M+R_{k-1}+1$ $M+R_k)$ where $M+1$ is the minimum income to save and 'i' units are being saved in the income group $(M+R_{i-1}+1,\ M+R_i)$.

In particular M was taken to be 6 units and $R_0=0$, $R_1=2$, $R_2=4$, *i.e.* $R_i=2i$ and the maximum income to be 14 units.

Stable probabilities Pi's were estimated from these equations by Monte-Carlo method of estimation. The stable probabilities thus estimated are given below for certain x^2-distributions for different degrees of freedom ($n's$).

n_x	P_1	P_2	P_3	P_4	P_5
3	·911	·059	·020	·006	·004
4	·802	·100	·052	·024	·022
7	·438	·192	·129	·085	·156
16	·098	·084	·096	·103	·625
20	·054	·058	·069	·089	·730
25	·042	·043	·055	·070	·790
30	·020	027	·044	·050	·859


Y →
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
P_0
P_1
P_2
P_3
P_4
STABLE PROBABILITIES
INDEX
χ^2_3
χ^2_4
χ^2_7
χ^2_{16}
χ^2_{20}
χ^2_{25}
χ^2_{30}

From the graph attached, it is evident that the probability for higher savings is increasing as the income distribution follows a x^2-distribution with large degrees of freedom.

The increasing nature of $E(x^2_n)$ as n increases makes the probability for high cumulative savings to become high.

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'n'	o	α	β
3	·5254	·428571	·000044
4	·5845	·256308	·0102313
5	·5850	·09766	·051516
6	·6070	·001600	·016180
7	·73350	·096778	·00000034
8	·7597	·075948	·00000084
9	·7800	·061213	·000134
10	·7992	·05042	·0000002346
11	·8140	·042251	·000906
12	·8275	·035926	·0000029
13	·8389	·030926	·0000016
14	·8389	·026420	·000356
15	·88576	·023621	·0000017
16	·8753	·02097	·0000045
17	·8735	·018638	·000000137
18	·8788	·18713	·00000010
19	·8845	·015076	·0000004
20	·8920	·01364	·000032
21	·8220	·012446	·00001002
22	·8988	·01138	·00000047
23	·9029	·01045	·00000056
24	·90816	·09616	·0000159
25	·9099	·008901	·00000094
26	·9145	·008244	·000011
27	·9160	·007679	·000001
28	·9180	·007148	·0000075
29	·9215	·005687	·00000023
30	·9239	·0062562	·00000046

APPENDIX

The two parameter discrete approximation to any unimodal continuous distribution developed by M.P. Sastry is

$$p_i = i[\alpha(\theta) + i\beta(\theta)]s\theta_i$$

$$i = 0, 1, 2, \ldots\ldots\ldots\ldots$$

The discrete approximation to any x^2-distribution with 'n' degree of freedom can be formulated using the values of α, β and θ given below corresponding to each 'n' (degreee of freedom)